The effective internal magnetic field in nanomagnet
In which equilibrium magnetization direction is in-plane

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Case when **equilibrium magnetization of the nanomagnet is in-plane**

(Conditions) A very strong exchange interaction between all localized electrons. As a result, there are no magnetic domains and all spins are parallel each other

(used Fact) localized electrons at interface and in the bulk experience different magnetic field of the spin-orbit interaction. However, their spins are strongly bound together and all spin can be considered as one magnetic object, which experience one common effective field, which is called the internal magnetic field

(object): a nanomagnet, which easy axis is perpendicular-to-plane.

The magnetization is aligned along the direction of the total magnetic field, which is some of the internal and external magnetic field $H_{total}=H_{int}+H_{ext}$. Therefore, magnetization angle β_M with respect to film normal can be calculated as

$$\tan\left(\beta_M\right) = \frac{M_\perp}{M_{||}} = \frac{H_{\mathrm{ext},\perp} + H_{\mathrm{int},\perp}}{H_{\mathrm{ext},||} + H_{\mathrm{int},||}} \quad (1)$$

in equilibrium (in absence of external magnetic field) $\alpha_M=0$
Taking into account that the magnitude of the magnetization can be calculated as

$$M^2 = M_\perp^2 + M_{||}^2 \quad (2)$$

The Eq.(1) is simplified as

$$\frac{\frac{M_\perp}{M}}{\sqrt{1-\left(\frac{M_\perp}{M}\right)^2}} = \frac{H_{\mathrm{ext},\perp} + H_{\mathrm{int},\perp}}{H_{\mathrm{ext},||} + H_{\mathrm{int},||}} \quad (3)$$

It assumed that the nanomagnet is sufficiently wide (in comparison to its thickness), so there is neither demagnetization field nor the spin-orbit field in the in--plane direction. As a result, the in-plane internal magnetic field has only one contribution of magnetic field induced by the magnetization, which is just the magnetic field along the magnetization:

$$H_{\mathrm{int},\|} = M_{\|} = M\sqrt{1-\left(\frac{M_{\perp}}{M}\right)^2} \quad (4)$$

Substitution of Eq.(4) into (3) gives

$$\frac{\frac{M_{\perp}}{M}}{\sqrt{1-\left(\frac{M_{\perp}}{M}\right)^2}} = \frac{H_{\mathrm{ext},\perp} + H_{\mathrm{int},\perp}}{H_{\mathrm{ext},\|} + M\sqrt{1-\left(\frac{M_{\perp}}{M}\right)^2}} \quad (5)$$

The perpendicular-to-plane component of magnetization $M_{\perp}$ is calculated from known (measured) anisotropy field H_{ani}. the in--plane component of the magnetization linearly The dependence is linear and the proportionality coefficient is defined as the anisotropy field:

$$\frac{M_{\perp}}{M} = \frac{H_{\mathrm{ext},\perp}}{H_{ani}} \quad (6)$$

From Eq (6) the perpendicular-to-plane component of the magnetization can be calculated as:

$$M_{\perp} = M\frac{H_{\mathrm{ext},\perp}}{H_{ani}} \quad (6a)$$

Substitution of Eq.(6a) into Eq.(5) gives

$$\frac{\frac{H_{\mathrm{ext},\perp}}{H_{ani}}}{\sqrt{1-\left(\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\right)^2}} = \frac{H_{\mathrm{ext},\perp} + H_{\mathrm{int},\perp}}{H_{\mathrm{ext},\|} + M\sqrt{1-\left(\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\right)^2}} \quad (7)$$

From Eq.(7) the perpendicular component of the internal magnetic field is found as

$$H_{\mathrm{int},\perp} = \frac{\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\left[H_{\mathrm{ext},\|} + M\sqrt{1-\left(\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\right)^2}\right]}{\sqrt{1-\left(\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\right)^2}} - H_{\mathrm{ext},\perp} \quad (8)$$

or

$$H_{\mathrm{int},\perp} = \frac{H_{\mathrm{ext},\perp}}{H_{ani}} \left[M + \frac{H_{\mathrm{ext},\parallel}}{\sqrt{1-\left(\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\right)^2}} \right] - H_{\mathrm{ext},\perp} \quad (8a)$$

or

$$H_{\mathrm{int},\perp} = H_{\mathrm{ext},\perp} \left[\frac{M}{H_{ani}} + \frac{H_{\mathrm{ext},\parallel}}{H_{ani}\sqrt{1-\left(\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\right)^2}} - 1 \right] \quad (8b)$$

From Eqs.(6) and (4a)

$$H_{\mathrm{int},\parallel} = M\sqrt{1-\left(\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\right)^2} \quad (4)$$

Magnitude of the internal magnetic field

$$H_{\mathrm{int}} = \sqrt{H_{\mathrm{int},\parallel}^2 + H_{\mathrm{int},\perp}^2} = \sqrt{M^2\left(1-\left(\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\right)^2\right) + H_{\mathrm{ext},\perp}^2 \left[\frac{M}{H_{ani}} + \frac{H_{\mathrm{ext},\parallel}}{H_{ani}\sqrt{1-\left(\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\right)^2}} - 1 \right]^2} \quad (19)$$

The total magnetic field

The total magnetic field is the sum of the internal and external magnetic fields.
From Eq.(4)

$$H_{\mathrm{total},\parallel} = H_{\mathrm{int},\parallel} + H_{\mathrm{ext},\parallel} = H_{\mathrm{ext},\parallel} + M\sqrt{1-\left(\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\right)^2} \quad (20)$$

from Eq.(8a)

$$H_{\mathrm{total},\perp} = H_{\mathrm{int},\perp} + H_{\mathrm{ext},\perp} = H_{\mathrm{ext},\perp} \left[\frac{M}{H_{ani}} + \frac{H_{\mathrm{ext},\parallel}}{H_{ani}\sqrt{1-\left(\frac{H_{\mathrm{ext},\perp}}{H_{ani}}\right)^2}} \right] \quad (21)$$

The absolute value of the total effective magnetic field

$$H_{\text{total}} = \sqrt{H_{\text{total},\perp}^2 + H_{\text{total},||}^2} =$$

$$\sqrt{\left[H_{\text{ext},||} + M\sqrt{1-\left(\frac{H_{\text{ext},\perp}}{H_{ani}}\right)^2}\right]^2 + H_{\text{ext},\perp}^2\left[\frac{M}{H_{ani}} + \frac{H_{\text{ext},||}}{H_{ani}\sqrt{1-\left(\frac{H_{\text{ext},\perp}}{H_{ani}}\right)^2}}\right]^2} \qquad (22)$$

when there is no external in-plane magnetic field $H_{\text{ext},||} = 0$

$$H_{\text{total}} = \sqrt{M^2\left(1-\left(\frac{H_{\text{ext},\perp}}{H_{ani}}\right)^2\right) + H_{\text{ext},\perp}^2\frac{M^2}{H_{ani}^2}} = M \qquad (22)$$

when there is no perpendicular-to-lane field $H_{\text{ext},\perp} = 0$

$$H_{\text{total}} = \sqrt{\left[H_{\text{ext},||} + M\right]^2} = H_{\text{ext},||} + M \qquad (22)$$