Strength of Perpendicular Magnetic Anisotropy (PMA). Magnetization tilt under an in-plane magnetic field

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The strength of perpendicular magnetic anisotropy (PMA) is calculated for a homogeneous thin ferromagnetic film. The PMA strength is defined by the balance between two opposing magnetic fields. The magnetic field H_{so} of the spin–orbit interaction, which is strongest at the film interface, forces the magnetization to align perpendicular to the film surface. In contrast, the demagnetization field, directed opposite to the spin–orbit field H_{so}, forces the in-plane magnetization alignment. In a film with PMA, the overall magnetic field of the spin–orbit interaction is larger than the demagnetization field. The main part of this work was completed in March 2018

I. CONDITIONS, ASSUMPTIONS, AXIS DIRECTIONS

(geometry and material): The static magnetic properties of a ferromagnetic film are calculated. The normal of the boundary plane is along the z- direction. There are no boundaries in the x- and y- direction. The exchange interaction in the film is strong enough so that all spins are aligned in the same direction for all calculated conditions.

(condition):The equilibrium magnetization is perpendicular to the plane (the z- direction). It is the case of a relatively thin ferromagnetic field, when the interfacial anisotropy overcomes the demagnetization field.

(Magnetic fields affecting magnetization):

(*field 1*): Magnetic field created by the spins: $\vec{H}_M = \vec{M}$. (*field 2*): Demagnetization field: $H_{demag,z} = -k_{demag} \cdot M_z$. (*field 3*): Magnetic field of spin-orbit interaction: $\vec{H}_{so} = \hat{k}_{so}\vec{H}_{total}$.

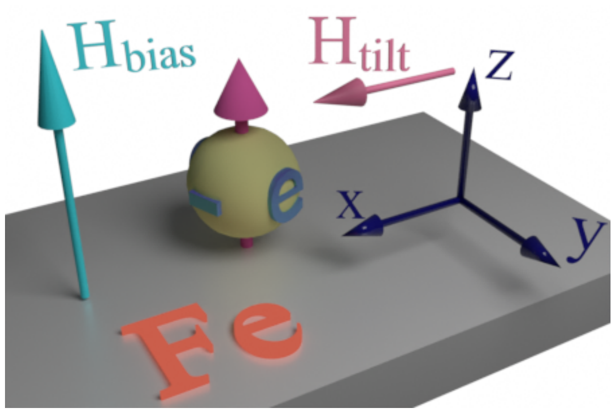

FIG. 1. Ferromagnetic iron film with perpendicular magnetic anisotropy (PMA): geometry. The green ball shows the equilibrium spin direction along the easy axis, which is oriented normal to the film plane (along the z-axis). The strength of the magnetic anisotropy is probed by applying a tilt field H_{tilt} perpendicular to the equilibrium spin direction (along the x-axis). The field H_{tilt} forces the spins to tilt away from the easy axis; the stronger the magnetic anisotropy, the larger the value of HH_{tilt} required to achieve the same tilt angle. A bias magnetic field H_{bias} is applied along the easy axis (the z-axis) to enhance the effective anisotropy field.

II. SUMMARY

(Fact) Due to magnetic anisotropy, the magnetization is aligned along the easy axis. To evaluate the strength of magnetic anisotropy, an external magnetic field is applied perpendicular to the easy axis. The measured response—how strongly the magnetization resists being pulled away from the easy axis—reveals the anisotropy strength.

When such a perpendicular field is applied, the magnetization tilts away from the easy axis. A fundamental property of magnetic materials, originating from spin-orbit interaction and the demagnetization field, is that the perpendicular component of the magnetization (the in-plane component, M_x, in this case) is linearly proportional to the applied field M_x (See Eq. 22):

$$\frac{M_x}{M} = \frac{H_x}{H_{ani}} \tag{1}$$

The proportionality constant is known as the anisotropy field H_{ani}, which quantitatively defines the strength of the magnetic anisotropy.

In absence of a bias magnetic field ($H_z = 0$) the anisotropy field is calculated as (Eq.(19)):

$$H^0_{ani} = 2M[(1 + k_{so})(1 - k_{demag}) - 1] \tag{2}$$

wherek_{so} is the coefficient of spin-orbit interaction and k_{demag} is the coefficient of demagnetization. When the spin-orbit interaction becomes stronger and k_{so} increases, the anisotropy field increases. When the surface becomes smoother and k_{demag} increases approaching 1, the anisotropy field decreases. It should be noted that a smoother interface enhances the strength of spin- orbit interaction as well. Therefore, the smoother interface can either increase or decrease the anisotropy field depending on the balance between two contributions (See Ref.).

In the case when the bias magnetic field H_z is applied along the easy axis, the anisotropy field is calculated as (See Eq. 25)

$$H_{ani} = H^0_{ani} + H_z + k_{so} H_z \tag{3}$$

The second term in the equation corresponds to the bulk contribution, representing the alignment of magnetization along H_z, which doesn't provide informative insights. Consequently, when conducting data analysis, it is more advantageous to utilize the relationship between $H_{ani} - H_z$, and H_z as

$$H_{ani} - H_z = H^0_{ani} + k_{so} H_z \tag{4}$$

This approach ensures that the significant bulk contribution does not obscure a potentially weaker dependence on k_{so}.

In a ferromagnetic film, the spins do not align exactly with the applied magnetic field. Instead, they tend to tilt away from the field direction toward the easy magnetic axis. The relationship between the angle of the applied magnetic field α_H, and the resulting tilt angle of the magnetization α_M is given by (see Eq. 35):

$$\frac{1}{\tan(\alpha_M)} = \frac{H^0_{ani}}{H} \frac{\cos(\alpha_M)}{sin(\alpha_H)} + \frac{1 + k_{so}}{\tan(\alpha_H)} \tag{5}$$

III. CALCULATIONS OF ANISOTROPY FIELD

A. Minimizing of magnetic energy

The equilibrium magnetization direction of a nanomagnet is the direction at which magnetic energy is the smallest. The calculation of magnetic energy is as follows:

$$-E = \vec{H}_{total} \cdot \vec{M} = (\vec{H} + \vec{M} + \vec{H}_{demag} + \vec{H}_{so}) \cdot \vec{M} \tag{6}$$

where $\vec{M}$ is the magnetization, $\vec{H}_{total}$ is the total magnetic field inside the nanomagnet, $\vec{H}_{demag}$ is the demagnetization field and $\vec{H}_{so}$ is the magnetic field of spin- orbit interaction (SO).

The magnetic field $\vec{H}_{so}$ is directly related to the total magnetic field experienced by an electron. In the presence of an anisotropy, $\vec{H}_{so}$ can be described using the tensor $\hat{k}_{so}$ as:

$$\vec{H}_{so} = \hat{k}_{so}(\vec{H} + \vec{M} + \vec{H}_{demag}) \tag{7}$$

The z-axis is established as perpendicular to the plane, while the x-axis is set within the plane. In the case of an amorphous nanomagnet, anisotropy in spin-orbit interaction can occur only between the z- and x-axes. Consequently, the tensor $\hat{k}_{so}$ can be written as follows:

$$\hat{k}_{so} = \begin{pmatrix} k_{so,x} & 0 & 0 \\ 0 & k_{so,x} & 0 \\ 0 & 0 & k_{so,z} \end{pmatrix} \tag{8}$$

The demagnetization field H_{demag} is always directed along the z-axis and is proportional to z- component of magnetization M_z

$$H_{demag,z} = -k_{demag} \cdot M_z \tag{9}$$

Rewriting Eq. 6 in components gives

$$\begin{aligned} -E = M_x \cdot (H_x + M_x + H_{so,x}) + \\ + M_z \cdot (H_z + M_z + H_{demag} + H_{so,z}) \end{aligned} \tag{10}$$

Substitution of Eqs. 7,8,9 into Eq. 10 gives

$$\begin{aligned} -E = M_x \cdot (1 + k_{so,x})(H_x + M_x) + \\ + M_z \cdot (1 + k_{so,z})[H_z + (1 - k_{demag}) \cdot M_z] \end{aligned} \tag{11}$$

The effective coefficient k_{so} of SO is defined as:

$$1 + k_{so} = \frac{1 + k_{so,z}}{1 + k_{so,x}} \tag{12}$$

When the spin–orbit interaction is stronger in the direction perpendicular to the film plane than in the in-plane direction, k_{so} is positive. Conversely, when the spin–orbit interaction is stronger in the in-plane direction than in the perpendicular-to-film direction, k_{so} is negative.

Substitution of Eq. 12 into Eq. 11 gives

$$\begin{aligned} \tfrac{-E}{1+k_{so,x}} = M_x(H_x + M_x) + \\ + M_z \cdot (1 + k_{so})[H_z + (1 - k_{demag}) \cdot M_z] \end{aligned} \tag{13}$$

Under an applied external magnetic field the magnetization M is tilted, but does not change its value. The ratio between two components of M is:

$$M_z = \sqrt{M^2 - M_x^2} \tag{14}$$

Substitution of Eq. 14 into Eq. 13 gives

$$\begin{aligned} \tfrac{-E}{1+k_{so,x}} = M^2 + M_x H_x + H_z(1 + k_{so})\sqrt{M^2 - M_x^2} + \\ + (M^2 - M_x^2)[(1 + k_{so})(1 - k_{demag}) - 1] \end{aligned} \tag{15}$$

The equilibrium magnetization angle corresponds to the orientation where the magnetic energy is at its minimum. The minimum can be found from the condition:

$$\frac{\partial E}{\partial M_x} = 0 \tag{16}$$

B. Anisotropy field in absence of bias magnetic field

When there is no perpendicular external magnetic field ($H_z = 0$), the condition specified in Eq.16 yields:

$$H_x - 2M_x[(1+k_{so})(1-k_{demag})-1] = 0 \tag{17}$$

Solution of Eq. 17 gives the linear relation between M_x and H_x as:

$$\frac{M_x}{M} = \frac{H_x}{H^0_{ani}} \tag{18}$$

where the anisotropy field H^0_{ani} in absence of the external field is calculated as:

$$H^0_{ani} = 2M[(1+k_{so})(1-k_{demag})-1] \tag{19}$$

C. anisotropy field in presence of bias magnetic field

Substitution of Eq. 19 into Eq. 15 gives:

$$\frac{-E}{1+k_{so,x}} = M^2 + M_x H_x + H_z(1+k_{so})\sqrt{M^2 - M_x^2} + \\ +(M^2 - M_x^2)H^0_{ani}/(2M) \tag{20}$$

Substitution of Eq. 20 into the condition 16 gives:

$$H_x \cdot (1 - \tfrac{M_x}{M} H^0_{ani}) - H_z(1+k_{so})\frac{M_x}{\sqrt{M^2-M_x^2}} = 0 \tag{21}$$

The solution of Eq. 20 is similar to the solution 19 and can be express as:

$$\frac{M_x}{M} = \frac{H_x}{H_{ani}} \tag{22}$$

where the anisotropy field H_{ani} is calculated as:

$$H_{ani} = H^0_{ani} + H_z \frac{1+k_{so}}{\sqrt{1-\frac{M_x^2}{M^2}}} \tag{23}$$

The following condition holds true for small to moderate tilting angles:

$$\frac{M_x^2}{M^2} \ll 1 \tag{24}$$

Thus, even in the presence of an external magnetic field, there is a linear relation between M_x and H_x. The anisotropy field H_{ani} is calculated from Eq. 23 as:

$$H_{ani} = H^0_{ani} + H_z + k_{so} H_z \tag{25}$$

Equation (25) determines the anisotropy field in the presence of a bias magnetic field H_z. It represents the main result of this article.

D. Internal magnetic field in a ferromagnet

Within a nanomagnet, an internal magnetic field H_{int} maintains the magnetization along its easy axis even without the external magnetic field H_z. Given that the internal and external magnetic fields share the same nature, their impact on the anisotropy field should be analogous. This implies that H_{ani} should solely depend on $H_{int} + H_z$. Noting that, Eq. 25 can be reformulated as:

$$H_{ani} = (H_z + H_{int}) + k_{so}(H_z + H_{int}) \tag{26}$$

where

$$H^0_{ani} = H_{int} + k_{so} H_{int} \tag{27}$$

The internal magnetic field H_{int} can be calculated from Eq. 27 as:

$$H_{int} = \frac{H^0_{ani}}{1+k_{so}} \tag{28}$$

It should be noted that when the external magnetic field H_z is applied opposite to the magnetization and its magnitude equals H_{int}, the effective magnetic anisotropy vanishes ($H_{ani} = 0$).

Additionally, it should be noted that electrons at the interface and in the bulk experience substantially different internal magnetic fields due to the differing strengths of the spin–orbit interaction acting on them. Therefore, H_{int} should be regarded as an effective average parameter used for practical calculations.

Furthermore, it should be noted that Eq. 28 describes the internal magnetic field only in the case where the external magnetic field is applied strictly along the easy axis and the magnetization remains aligned with it. When the magnetization is tilted away from the easy axis, both the magnitude and direction of the internal magnetic field are altered.

E. Tilting angle of magnetization under an external magnetic field applied perpendicularly to the easy axis

In vacuum, spins align precisely with the external magnetic field. In contrast, in a ferromagnetic film, spins do not align exactly with the applied magnetic field due to the influence of two additional internal magnetic fields: the demagnetizing field and the magnetic field of spin–orbit interaction. In the following, the angle between the applied magnetic field and the spin direction is calculated.

The tilt angle α_M of the magnetization relative to the easy axis (the z-axis) is given by

$$\sin(\alpha_M) = \frac{M_\perp}{M} = \frac{M_x}{M} \tag{29}$$

Taking into account Eq. 1 , Eq. 29 becomes

$$\sin(\alpha_M) = \frac{H_x}{H_{ani}} \tag{30}$$

Taking into account Eq. 23, Eq. 30 becomes

$$\frac{1}{\sin(\alpha_M)} = \frac{H^0_{ani}}{H_x} + \frac{(1+k_{so})H_z}{H_x\sqrt{1-\frac{M_x^2}{M^2}}} \tag{31}$$

The angle α_H, at which the magnetic field is applied, can be calculated as

$$\tan(\alpha_H) = \frac{H_x}{H_z} \tag{32}$$

From Eq.29

$$\sqrt{1-\frac{M_x^2}{M^2}} = \frac{M_z}{M} = \cos(\alpha_M) \tag{33}$$

Substitution of Eqs. 33 and 32 into Eq. 31 and using $H_x = H \cdot sin(\alpha_H)$ gives

$$\frac{1}{\sin(\alpha_M)} = \frac{H^0_{ani}}{H}\frac{1}{sin(\alpha_H)} + \frac{1+k_{so}}{\cos(\alpha_M)\cdot\tan(\alpha_H)} \tag{34}$$

where

From Eq. 34, the **final expression** describing the relationship between the angle of the applied magnetic field α_H and the resulting tilt angle α_M of the magnetization is obtained as follows:

$$\frac{1}{\tan(\alpha_M)} = \frac{H^0_{ani}}{H}\frac{\cos(\alpha_M)}{sin(\alpha_H)} + \frac{1+k_{so}}{\tan(\alpha_H)} \tag{35}$$

In the vacuum ($k_{so} = 0$ and $H^0_{ani} = 0$), the magnetization is aligned along the external magnetic field:

$$\frac{1}{\tan(\alpha_M)} = \frac{1}{\tan(\alpha_H)} \tag{36}$$